# Assessment Schedule – 2007

## Scholarship Mathematics with Calculus (93202)

### Evidence Statement

For up to a maximum of 4 question parts a single minor error in that part was accepted without loss of marks.

| Question | Evidence | Code | Judgement |
|---|---|---|---|
| **ONE (a)** | $V = \int_0^h \pi y^2 \mathrm{d}x$<br>$= \int_0^h \pi 4ax \mathrm{d}x$<br>$= 4a\pi \left[ \frac{x^2}{2} \right]_0^h$<br>$= 2a\pi h^2$ | **2** | |
| ONE (b) | $\text{SA} = \int_0^h y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\mathrm{d}x$<br>$= \int_0^h \sqrt{4ax}\sqrt{1+\frac{4a^2}{y^2}}\mathrm{d}x$<br>$= \int_0^h \sqrt{4ax}\sqrt{1+\frac{4a^2}{4ax}}\mathrm{d}x$<br>$= \int_0^h \sqrt{4ax+4a^2}\mathrm{d}x$ | **3** | Accept working with $2\pi$ throughout<br><br>Accept up to $\sqrt{4ax}\sqrt{1+\frac{a}{x}}$ |
| | **EITHER**<br>$= \int_0^h 2\sqrt{a}\sqrt{x+a}\mathrm{d}x$<br>$= 2\sqrt{a}\left[\frac{2}{3}(x+a)^{\frac{3}{2}}\right]_0^h$<br>$= \frac{4\sqrt{a}}{3}\left[(h+a)^{\frac{3}{2}} - (a)^{\frac{3}{2}}\right]$<br>$= \frac{4}{3}\left[\sqrt{a}(h+a)^{\frac{3}{2}} - a^2\right]$<br>**OR**<br>$= \left[\frac{2}{12a}\left(4ax+4a^2\right)^{\frac{3}{2}}\right]_0^h$<br>$= \frac{1}{6a}\left[\left(4ah+4a^2\right)^{\frac{3}{2}} - \left(4a^2\right)^{\frac{3}{2}}\right]$<br>$= \frac{1}{6a}\left[\left(4ah+4a^2\right)^{\frac{3}{2}} - 8a^3\right] = \frac{4}{3a}\left[\left(ah+a^2\right)^{\frac{3}{2}} - a^3\right]$<br>$= \frac{4}{3}\left[\sqrt{a}(h+a)^{\frac{3}{2}} - a^2\right]$ | **1** | Simplification not required |

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| **ONE (c)** | $\frac{\mathrm{d}h}{\mathrm{d}t} = 0.26 \text{ mm / s when } h = 8a$<br><br>$V = 2\pi a h^2$<br>$\frac{\mathrm{d}V}{\mathrm{d}h} = 4\pi a h$<br><br>$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h}\frac{\mathrm{d}h}{\mathrm{d}t}$<br>$= 4\pi a h \frac{\mathrm{d}h}{\mathrm{d}t}$<br>$= 4\pi a.8a.(-0.26) \text{ at } h = 8a$<br>$= -8.32\pi a^2$ | **3**<br><br>**1** | Accept +0.26 etc throughout |
| | $\frac{\mathrm{d}V}{\mathrm{d}t}\,\alpha\, SA$<br>$\frac{\mathrm{d}V}{\mathrm{d}t} = k\left(\frac{1}{6a}\left[\left(4ah + 4a^2\right)^{\frac{3}{2}} - 8a^3\right]\right)$<br>$h = 8a$<br>$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{k}{6a}\left[\left(4ah + 4a^2\right)^{\frac{3}{2}} - 8a^3\right] = -8.32\pi a^2$<br>$\frac{k}{6a}\left[\left(36a^2\right)^{\frac{3}{2}} - 8a^3\right] = -8.32\pi a^2$<br>$\frac{k}{6a}\left[216a^3 - 8a^3\right] = -8.32\pi a^2$<br>$\frac{k208a^3}{6a} = -8.32\pi a^2$<br>$k = -0.24\pi \; (= -0.754)$ | **2** | Accept –0.12 |
| | $\frac{\mathrm{d}V}{\mathrm{d}t} = -0.24\pi\left[\frac{1}{6a}\left(4ah + 4a^2\right)^{\frac{3}{2}} - 8a^3\right]$<br>$h = 3a$<br>$\frac{\mathrm{d}V}{\mathrm{d}t} = -0.24\pi\left[\frac{1}{6a}\left(12a^2 + 4a^2\right)^{\frac{3}{2}} - 8a^3\right]$<br>$= -\frac{0.24\pi}{6a}\left(64a^3 - 8a^3\right)$<br>$= -2.24\pi a^2 (= -7.037a^2)$ | | Accept $-1.12a^2$ |

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| **TWO (a)** | $10\pi\cos\left(\frac{\pi x}{2}\right)+10\pi\cos\left(\frac{2\pi x}{3}\right)=0$<br>$10\pi\left(\cos\left(\frac{\pi x}{2}\right)+\cos\left(\frac{2\pi x}{3}\right)\right)=0$<br>$2\cos\left(\frac{7\pi x}{12}\right)\cos\left(\frac{\pi x}{12}\right)=0$<br>$\cos\left(\frac{7\pi x}{12}\right)=0$ or $\cos\left(\frac{\pi x}{12}\right)=0$<br><br>$\frac{7\pi x}{12}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},\frac{9\pi}{2}.....$<br>$x=\frac{6}{7},\frac{18}{7},\frac{30}{7},6,\frac{54}{7}...$<br>$(=0.86,2.57,4.29,6,7.7,...)$<br>or $\frac{\pi x}{12}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},.....$<br>$x=6,18,30,...$<br><br>For $0\le x\le 2\pi,\ x=\frac{6}{7},\frac{18}{7},\frac{30}{7},6$<br>$(=0.86,2.57,4.29,6)$ | **2** | |
| **TWO (b)(i)** | **EITHER**<br>By Ratio<br><br>ARM: 4 min: $2\pi$, 1 min: $\frac{2\pi}{4}$, $t$ min: $\frac{\pi t}{2}$<br>WHEEL: 3 min: $2\pi$, 1 min: $\frac{2\pi}{3}$, $t$ min: $\frac{2\pi t}{3}$<br><br>**OR**<br>by Calculus<br>$\frac{d\theta}{dt}=\frac{2\pi}{4}$ radians / min    $\frac{d\alpha}{dt}=\frac{2\pi}{3}$ radians / min<br>$\theta=\frac{\pi}{2}t$    $\alpha=\frac{2\pi}{3}t$<br><br>Hence the required function is:<br><br>$h(t)=35+20\sin\left(\frac{\pi}{2}t\right)+15\sin\left(\frac{2\pi}{3}t\right)$ | **3** | |

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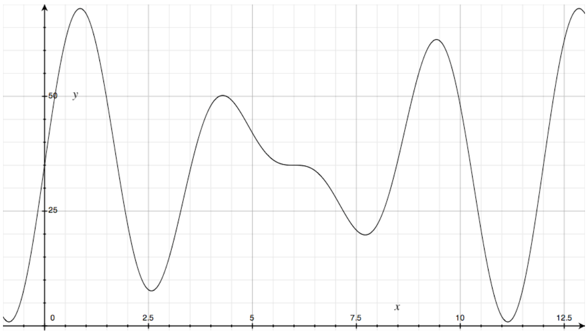
| **TWO (b)(ii)** | $h'(t) = 20\frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right) + 15\frac{2\pi}{3}\cos\left(\frac{2\pi}{3}t\right)$<br>$= 10\pi\cos\left(\frac{\pi}{2}t\right) + 10\pi\cos\left(\frac{2\pi}{3}t\right) = 0$ for max and min<br><br>From part (a)<br><br>$t = \frac{6}{7}, \frac{18}{7}, \frac{30}{7}, 6$<br>$(= 0.86, 2.57, 4.29, 6)$<br>and also<br>$t = \frac{54}{7}, \frac{66}{7}, \frac{78}{7}, 12$<br>$(= 7.71, 9.43, 11.14, 12)$<br>The graph is:<br> | **3** | |
| | For $t = \frac{6}{7}$ (0.86..) $\quad h\left(\frac{6}{7}\right) = 69.1224\ldots$<br>$h''(t) = -5\pi^2\sin\left(\frac{\pi}{2}t\right) - \frac{20}{3}\pi^2\sin\left(\frac{2\pi}{3}t\right)$ and<br>$h''\left(\frac{6}{7}\right) = -5\pi^2\sin\left(\frac{3\pi}{7}\right) - \frac{20}{3}\pi^2\sin\left(\frac{4\pi}{7}\right) < 0$ (using sin and quadrants) and hence a local maximum | **1 derivative test for maximum** | |
| | For $t = \frac{30}{7}$ (4.29) $\quad h\left(\frac{30}{7}\right) = 50.186$<br>$h''\left(\frac{30}{7}\right) = -5\pi^2\sin\left(\frac{15\pi}{7}\right) - \frac{20}{3}\pi^2\sin\left(\frac{20\pi}{7}\right) < 0$<br>For $t = \frac{66}{7}$ (9.43) $\quad h\left(\frac{66}{7}\right) = 62.364$<br>$h''\left(\frac{66}{7}\right) = -5\pi^2\sin\left(\frac{33\pi}{7}\right) - \frac{20}{3}\pi^2\sin\left(\frac{44\pi}{7}\right) < 0$<br>and since the function is continuous and there are no other local maximum points in the domain the maximum point is $\left(\frac{6}{7}, 69.1\right)$. | **1 for checking to $t$=4π (or 12)** | |
| | **OR**<br>Using $h'$ for testing maxima<br>before $h'\left(\frac{5}{7}\right) = 15.978 > 0$<br>after $h'(1) = -15.7 < 0$<br>Hence max $\left(\frac{6}{7}, 69.1\right)$<br>etc and Maximum Height above the ground is 69 m (2s.f.) | **1, correct answer** | |

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| **THREE (a)** | $h'(x) = -2\sin x \quad f'(x) = 6\cos(2x)$<br>$6\cos(2x) = -2\sin x$<br>$3(1-2\sin^2 x) = -\sin x$<br>$6\sin^2 x - \sin x - 3 = 0$<br>$\sin x = \dfrac{1 \pm \sqrt{1+72}}{12}$<br>$= \dfrac{1 \pm \sqrt{73}}{12}$ | **2** | |
| | $\sin x = 0.7953$ or $\sin x = -0.6286$<br>$x = 0.9196$ or $x = \pi - 0.9196$ or $x = \pi + 0.6798$ or $x = 2\pi - 0.6798$<br>$x = 0.9196$ or $2.222$ or $3.8214$ or $5.6033$ (4s.f.) for $0 \le x \le 2\pi$ | **1** | Accept 2 correct for 1 mark |
| **THREE (b)** | Vertex $p(x)$ lies on $g(x) \Rightarrow (b,-c) \equiv (\pi,-2)$<br>$b = \pi \quad c = 2$ | **3** | |
| | EITHER: $p'(x) = f'(x)$ at intersection<br>So $-2a(x-\pi) = 6\cos 2x$<br>but $x = 2.575$<br>$a = \dfrac{-6\cos(5.15)}{2.575-\pi} = 2.244$ (4 s.f.)<br>OR:<br>$f(2.575) = 3\sin(2\times 2.575) = -2.717$<br>$p(2.575) = -a(2.575-\pi)^2 - 2$<br>So $-a(2.575-\pi)^2 - 2 = -2.717$<br>and $a = \dfrac{2+3\sin(2\times 2.575)}{-(2.575-\pi)^2}$<br>$= 2.234$ (4 s.f.) | **1**<br><br>**1 indep with *b, c*** | Accept equivalent |
| **THREE (c)** | $f(x)$ and $h(x)$ touch when their gradients are equal.<br>$f'(x) = h'(x)$<br>$6\cos(2x) = -2\sin x$<br>$x = 0.9196$ from part (b) | **3** | |
| | for $h(0.9196) = f(0.9196)$<br>$k + 2\cos 0.9196 = 3\sin(2\times 0.9196)$<br>$k = 3\sin(2\times 0.9196) - 2\cos 0.9196$<br>$= 1.6803$ | **1** | |
| | cuts twice for $k > 1.68$<br>Similarly the last touch is when $x = 3.8214$<br>$h(3.8214) = f(3.8214)$<br>$k + 2\cos 3.8214 = 3\sin(2\times 3.8214)$<br>$k = 3\sin(2\times 3.8214) - 2\cos 3.8214$<br>$= 4.4887$<br>When $k > 0$, cuts twice for $1.68 < k < 4.4887$ | **2** | Accept either case<br><br>Answers only – accept if correct to 2 d.p. |

<table>
<tr><td>FOUR (a)</td><td>

$$z^2=\frac{1}{2}\left(a+\sqrt{\left(a^2+b^2\right)}\right)-\frac{1}{2}\left(-a+\sqrt{\left(a^2+b^2\right)}\right)+2\mathrm{i}\sqrt{\frac{1}{4}\left(a^2+b^2-a^2\right)}$$

$$z^2=\frac{a}{2}+\frac{a}{2}+2\mathrm{i}\frac{|b|}{2}=a+\mathrm{i}|b|$$

</td><td>2</td><td>Accept with just $b$</td></tr>
<tr><td>FOUR (b)(i)</td><td>

$$f(0)=c=1+\frac{\mathrm{i}}{2}$$

$$f^2(0)=c^2+c=\left(1+\frac{\mathrm{i}}{2}\right)^2+\left(1+\frac{\mathrm{i}}{2}\right)$$

$$f^2(0)=\frac{7}{4}+\frac{3\mathrm{i}}{2}$$

and $\left|f^2(0)\right|=\frac{1}{4}\left(\sqrt{49+36}\right)=\frac{\sqrt{85}}{4}>2$, so $z=1+\frac{\mathrm{i}}{2}$ is not part of the set.

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$$f(0)=c=\mathrm{i}$$

$$f^2(0)=c^2+c=(\mathrm{i})^2+\mathrm{i}=-1+\mathrm{i}$$

and $\left|f^2(0)\right|=\sqrt{2}<2$, so OK so far.

$$f^3(0)=(-1+\mathrm{i})^2+\mathrm{i}=-2\mathrm{i}+\mathrm{i}=-\mathrm{i}$$

and $\left|f^3(0)\right|=1<2$, so OK so far.

$f^4(0)=(-\mathrm{i})^2+\mathrm{i}=-1+\mathrm{i}$ and so the process will loop continually with $\left|f^n(0)\right|<2$ for all $n$, and $z=\mathrm{i}$ is part of the set.

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<tr><td>FOUR (b)(ii)</td><td>

$$f(0)=c,\ f^2(0)=c^2+c=a+\mathrm{i}b+c$$

$$c=\sqrt{\frac{1}{2}\left(a+\sqrt{\left(4a^2\right)}\right)}+\mathrm{i}\sqrt{\frac{1}{2}\left(-a+\sqrt{\left(4a^2\right)}\right)}$$

$$c=\sqrt{\frac{1}{2}(3a)}+\mathrm{i}\sqrt{\frac{1}{2}(a)}=\sqrt{\frac{a}{2}}\left(\sqrt{3}+\mathrm{i}\right)$$

$$f^2(0)=a+\mathrm{i}b+\sqrt{\frac{a}{2}}\left(\sqrt{3}+\mathrm{i}\right)=a+\sqrt{\frac{3a}{2}}+\mathrm{i}\left(a\sqrt{3}+\sqrt{\frac{a}{2}}\right)$$

and when $a=\frac{1}{8}$, $f^2(0)=\frac{1}{8}+\sqrt{\frac{3}{16}}+\mathrm{i}\left(\frac{\sqrt{3}}{8}+\frac{1}{4}\right)$

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$$\frac{1+2\sqrt{3}}{8}+\mathrm{i}\left(\frac{\sqrt{3}+2}{8}\right)$$

so $\left|f^2(0)\right|=\frac{1}{8}\sqrt{\left(1+2\sqrt{3}\right)^2+\left(\sqrt{3}+2\right)^2}=\frac{1}{8}\sqrt{\left(13+4\sqrt{3}\right)+\left(7+4\sqrt{3}\right)}$

$$=\frac{1}{8}\sqrt{\left(20+8\sqrt{3}\right)}=\frac{1}{4}\sqrt{\left(5+2\sqrt{3}\right)}$$

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<tr><td>FIVE (a)</td><td>

EITHER

$x = at^2 + b \qquad y = 2at \qquad x = 3b - as^2 \qquad y = \sqrt{2}as$

$x - b = a\left(\frac{y}{2a}\right)^2 \qquad x - 3b = -a\left(\frac{y}{\sqrt{2}a}\right)^2$

$y^2 = 4a(x-b) \qquad y^2 = -2a(x-3b)$

Intersect where

$4a(x-b) = -2a(x-3b)$

$2x - 2b = -x + 3b$

$3x = 5b$

$x = \frac{5b}{3} \quad y^2 = 4a\left(\frac{2b}{3}\right) = \frac{8ab}{3}$

Points of intersection are $\left(\frac{5b}{3}, \pm 2\sqrt{\frac{2ab}{3}}\right)$

OR

Parametrically

$2at = \sqrt{2}as \qquad s = \sqrt{2}t$

when $at^2 + b = 3b - as^2$ substitute

$at^2 + b = 3b - 2at$

$3at^2 = 2b$

$t = \pm\sqrt{\frac{2b}{3a}} \qquad s = \pm 2\sqrt{\frac{b}{3a}}$

Points of intersection are $\left(\frac{2b}{3} + b, \pm 2a\sqrt{\frac{2b}{3a}}\right)$

$= \left(\frac{5b}{3}, \pm 2\sqrt{\frac{2ab}{3}}\right)$

</td><td>2</td><td></td></tr>
<tr><td>FIVE (b)</td><td>

EITHER

Ratio of the areas = $\dfrac{\cancel{2}\int_b^{\frac{5b}{3}} 2\sqrt{a}(x-b)^{\frac{1}{2}}\,dx}{\cancel{2}\int_{\frac{5b}{3}}^{3b} \sqrt{2}\sqrt{a}(3b-x)^{\frac{1}{2}}\,dx}$

Top = $2\sqrt{a}\left[\frac{2}{3}(x-b)^{\frac{3}{2}}\right]_b^{\frac{5b}{3}} = 2\sqrt{a} \times \frac{2}{3}\left(\frac{2b}{3}\right)^{\frac{3}{2}} = \frac{1}{6a}\left(\frac{8ab}{3}\right)^{\frac{3}{2}}$

Bottom = $\sqrt{2a}\left[\frac{-2}{3}(3b-x)^{\frac{3}{2}}\right]_{\frac{5b}{3}}^{3b} = -\sqrt{2a} \times \frac{2}{3}\left(-\frac{4b}{3}\right)^{\frac{3}{2}} = \frac{1}{3a}\left(\frac{8ab}{3}\right)^{\frac{3}{2}}$

Ratio = $\dfrac{\frac{1}{6a}\left(\frac{8ab}{3}\right)^{\frac{3}{2}}}{\frac{1}{3a}\left(\frac{8ab}{3}\right)^{\frac{3}{2}}} = \frac{3a}{6a} = \frac{1}{2}$

</td><td>3<br><br>1<br><br>1<br><br>1</td><td>Accept without 2's<br><br>Accept top or bottom<br><br>Accept 2:1<br><br>2 intns in $x$ correct but wrong limits, 1</td></tr>
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| | **OR**<br>Ratio of the areas = $\dfrac{\int_b^{\frac{5b}{3}} (4ax-4ab)^{\frac{1}{2}} dx}{\int_{\frac{5b}{3}}^{3b} (6ab-2ax)^{\frac{1}{2}} dx}$<br>Top = $\dfrac{1}{6a}\left[(4ax-4ab)^{\frac{3}{2}}\right]_b^{\frac{5b}{3}} = \dfrac{1}{6a}\left(\dfrac{8ab}{3}\right)^{\frac{3}{2}}$<br>Bottom = $-\dfrac{1}{3a}\left[(6ab-2ax)^{\frac{3}{2}}\right]_{\frac{5b}{3}}^{3b} = -\dfrac{1}{3a}\left(-\dfrac{8ab}{3}\right)^{\frac{3}{2}} = \dfrac{1}{3a}\left(\dfrac{8ab}{3}\right)^{\frac{3}{2}}$<br>Ratio $=\dfrac{1}{2}$<br>**OR**<br>For Curve 1 $\dfrac{\mathrm{dy}}{\mathrm{d}x} = \dfrac{4a}{2y} = \dfrac{2a}{y}$ so $\int_b^{\frac{5b}{3}} ydx = \int_0^{\sqrt{\frac{8ab}{3}}} y\dfrac{y}{2a}dy = \left[\dfrac{y^3}{6a}\right]_0^{\sqrt{\frac{8ab}{3}}} = \dfrac{1}{6a}\left(\dfrac{8ab}{3}\right)^{\frac{3}{2}}$<br>For Curve 2 $\dfrac{\mathrm{dy}}{\mathrm{d}x} = \dfrac{-2a}{2y} = -\dfrac{a}{y}$ so $\int_{\frac{5b}{3}}^{3b} ydx = \int_{\sqrt{\frac{8ab}{3}}}^{0} y\left(-\dfrac{y}{a}\right)dy = -\left[\dfrac{y^3}{3a}\right]_{\sqrt{\frac{8ab}{3}}}^{0} = \dfrac{1}{3a}\left(\dfrac{8ab}{3}\right)^{\frac{3}{2}}$<br>Ratio $=\dfrac{1}{2}$<br>**OR**<br>Translation of Curve 1<br>$y^2 = 4a\left(x-b+\dfrac{5b}{3}\right) = 4a\left(x+\dfrac{2b}{3}\right)$<br>$x = \dfrac{y^2}{4a} - \dfrac{2b}{3}$<br>Translation of Curve 2<br>$y^2 = -2a\left(x-3b+\dfrac{5b}{3}\right) = -2a\left(x-\dfrac{4b}{3}\right)$<br>$x = \dfrac{y^2}{-2a} + \dfrac{4b}{3}$<br>Ratio is<br>$-\int_0^{\sqrt{\frac{8ab}{3}}}\left(\dfrac{y^2}{4a}-\dfrac{2b}{3}\right)dy : \int_0^{\sqrt{\frac{8ab}{3}}}\left(\dfrac{y^2}{-2a}+\dfrac{4b}{3}\right)dy$<br>$-\left[\dfrac{y^3}{12a}-\dfrac{2by}{3}\right]_0^{\sqrt{\frac{8ab}{3}}} : \left[\dfrac{y^3}{-6a}+\dfrac{4by}{3}\right]_0^{\sqrt{\frac{8ab}{3}}}$<br>$-\left[\dfrac{\left(\frac{8ab}{3}\right)^{\frac{3}{2}}}{12a} - \dfrac{2b\left(\frac{8ab}{3}\right)^{\frac{1}{2}}}{3}\right] : \left[\dfrac{\left(\frac{8ab}{3}\right)^{\frac{3}{2}}}{-6a} + \dfrac{4b\left(\frac{8ab}{3}\right)^{\frac{1}{2}}}{3}\right]$<br>$\left[\dfrac{-\left(\frac{8ab}{3}\right)^{\frac{3}{2}} + 8ab\left(\frac{8ab}{3}\right)^{\frac{1}{2}}}{12a}\right] : \left[\dfrac{-2\left(\frac{8ab}{3}\right)^{\frac{3}{2}} + 16ab\left(\frac{8ab}{3}\right)^{\frac{1}{2}}}{12a}\right] = 1:2$<br>**OR**<br>Parametrically | | Accept 2:1 |

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| | Area 1<br>$\int y\mathrm{d}x = \int 2at\frac{\mathrm{d}x}{\mathrm{d}t}dt$     $y = 2at \quad x = at^2 + b$<br>$= 4a^2\int_0^p t^2\mathrm{d}t$     $\frac{\mathrm{d}y}{\mathrm{d}t} = 2a \quad \frac{\mathrm{d}x}{\mathrm{d}t} = 2at$<br>$= 4a^2\left[\frac{t^3}{3}\right]$     $at^2 + b = \frac{5b}{3}$<br>$= \frac{4a^2}{3}\frac{2b}{3a}\sqrt{\frac{2b}{3a}}$     $at^2 = \frac{2b}{3}$<br>$t^2 = \frac{2b}{3a}$<br><br>Area 2<br>$\int y\mathrm{d}x = \int_q^0 \sqrt{2}as(-2as)\mathrm{d}s$     $\frac{5b}{3} = 3b - as^2$<br>$= -2\sqrt{2}a^2\left[\frac{s^3}{3}\right]_q^0$     $s^2 = \frac{4b}{3a}$<br>$= \frac{2\sqrt{2}a^2}{3}\frac{4b}{3a}\sqrt{\frac{4b}{3a}}$<br><br>Ratio $= \frac{8ab}{9}\sqrt{\frac{2b}{3a}}\frac{9}{8\sqrt{2}ab}\sqrt{\frac{3a}{4b}}$<br>$= \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$ | | |
| **FIVE (c)** | <u>Curve 2</u><br>$x = 3b - as^2$     $y = \sqrt{2}as$<br>$\frac{\mathrm{d}x}{\mathrm{d}s} = -2as$     $\frac{\mathrm{d}y}{\mathrm{d}s} = \sqrt{2}a$<br>Point P: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}x} = \frac{\sqrt{2}a}{-2as} = \frac{\sqrt{2}}{-2s}$<br>Gradient of the Normal $= \frac{2s}{\sqrt{2}} = \sqrt{2}s$<br><br>Equation of the Normal is<br>$y - \sqrt{2}as = \sqrt{2}s\left(x - 3b + as^2\right)$<br>$y = 0 \quad -a = x - 3b + as^2$<br>Point Q $\left(3b - a - as^2, 0\right)$ | 3<br><br><br><br><br><br><br><br><br><br>1 | |
| | Midpoint PQ $\left(\frac{3b - a - as^2 + 3b - as^2}{2}, \frac{\sqrt{2}as + 0}{2}\right)$<br>$= \left(3b - \frac{a}{2} - as^2, \frac{as}{\sqrt{2}}\right)$ | | |

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| | $x = 3b - \frac{a}{2} - a\left(\frac{\sqrt{2}y}{a}\right)^2$<br>$\left(x - 3b + \frac{a}{2}\right) = \left(\frac{-2y^2}{a}\right)$<br>$y^2 = \frac{-a}{2}\left(x - 3b + \frac{a}{2}\right)$ | **2** | |
| | so $k = -\frac{1}{2}$, $m = \frac{1}{2}$, $n = -3$ | | |